6. Intermediate Representation

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Thanks to Jens Palsberg and Tony Hosking for their kind permission to reuse and adapt the CS132 and CS502 lecture notes.
http://www.cs.ucla.edu/~palsberg/
http://www.cs.purdue.edu/homes/hosking/

SSA lecture notes by Marcus Denker
Roadmap

- Intermediate representations
- Static Single Assignment
- SSA generation
- Dominance and SSA generation
- Applications of SSA
- \( \Phi \)-congruence and SSA removal

Roadmap

- Intermediate representations
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Why use intermediate representations?

1. Software engineering principle
   — break compiler into manageable pieces
2. Simplifies retargeting to new host
   — isolates back end from front end
3. Simplifies support for multiple languages
   — different languages can share IR and back end
4. Enables machine-independent optimization
   — general techniques, multiple passes
IR scheme

- front end produces IR
- optimizer transforms IR to more efficient program
- back end transforms IR to target code
Kinds of IR

- Abstract syntax trees (AST)
- Linear operator form of tree (e.g., postfix notation)
- Directed acyclic graphs (DAG)
- Control flow graphs (CFG)
- Program dependence graphs (PDG)
- Static single assignment form (SSA)
- 3-address code
- Hybrid combinations
Categories of IR

> Structural
  — graphically oriented (trees, DAGs)
  — nodes and edges tend to be large
  — heavily used on source-to-source translators

> Linear
  — pseudo-code for abstract machine
  — large variation in level of abstraction
  — simple, compact data structures
  — easier to rearrange

> Hybrid
  — combination of graphs and linear code (e.g. CFGs)
  — attempt to achieve best of both worlds
## Important IR properties

- Ease of generation
- Ease of manipulation
- Cost of manipulation
- Level of abstraction
- Freedom of expression (!)
- Size of typical procedure
- Original or derivative

Subtle design decisions in the IR can have far-reaching effects on the speed and effectiveness of the compiler!

» Degree of exposed detail can be crucial
Abstract syntax tree

An AST is a parse tree with nodes for most non-terminals removed.

Since the program is already parsed, non-terminals needed to establish precedence and associativity can be collapsed!

A linear operator form of this tree (postfix) would be:

```
x 2 y * -
```
A DAG is an AST with unique, shared nodes for each value.

\[
\begin{align*}
x & := 2 \cdot y + \sin(2 \cdot x) \\
z & := x / 2
\end{align*}
\]
Control flow graph

> A CFG models *transfer of control* in a program
  —nodes are *basic blocks* (straight-line blocks of code)
  —edges represent *control flow* (loops, if/else, goto …)

```plaintext
if x = y then
  S1
else
  S2
end
S3
```
3-address code

> Statements take the form: \( x = y \text{ op } z \)
  — single operator and at most three names

\[
\begin{align*}
  x - 2 * y & \quad \Rightarrow \quad \text{t1} = 2 * y \\
  & \quad \text{t2} = x - \text{t1}
\end{align*}
\]

> Advantages:
  — compact form
  — names for intermediate values
### Typical 3-address codes

<table>
<thead>
<tr>
<th></th>
<th>assignments</th>
<th>branches</th>
<th>conditional branches</th>
<th>procedure calls</th>
<th>address and pointer assignments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$x = y \text{ op } z$</td>
<td>goto L</td>
<td>if $x \text{ relop } y$ goto L</td>
<td>param $x$</td>
<td>$x = &amp;y$</td>
</tr>
<tr>
<td></td>
<td>$x = \text{ op } y$</td>
<td></td>
<td></td>
<td>param $y$</td>
<td>$*y = z$</td>
</tr>
<tr>
<td></td>
<td>$x = y[i]$</td>
<td></td>
<td></td>
<td>call $p$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$x = y$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### 3-address code — two variants

<table>
<thead>
<tr>
<th>Quadruples</th>
<th>Triples</th>
</tr>
</thead>
<tbody>
<tr>
<td>x - 2 * y</td>
<td>x - 2 * y</td>
</tr>
<tr>
<td>(1) load t1 y</td>
<td>(1) load y</td>
</tr>
<tr>
<td>(2) loadi t2 2</td>
<td>(2) loadi 2</td>
</tr>
<tr>
<td>(3) mult t3 t2 t1</td>
<td>(3) mult (1) (2)</td>
</tr>
<tr>
<td>(4) load t4 x</td>
<td>(4) load x</td>
</tr>
<tr>
<td>(5) sub t5 t4 t3</td>
<td>(5) sub (4) (3)</td>
</tr>
</tbody>
</table>

- simple record structure
- easy to reorder
- explicit names
- table index is implicit name
- only 3 fields
- harder to reorder
IR choices

> Other hybrids exist
  — combinations of graphs and linear codes
  — CFG with 3-address code for basic blocks

> Many variants used in practice
  — no widespread agreement
  — compilers may need several different IRs!

> Advice:
  — choose IR with right level of detail
  — keep manipulation costs in mind
Roadmap

> Intermediate representations
> Static Single Assignment
> SSA generation
> Dominance and SSA generation
> Applications of SSA
> \(\Phi\)-congruence and SSA removal
SSA: Literature

Books:
- SSA Chapter in Appel
- Chapter 8.11 Muchnik

SSA Creation:
Cytron et al: *Efficiently computing Static Single Assignment Form and the Control Dependency Graph* (TOPLAS, Oct 1991)

> **Goal:** simplify procedure-global optimizations

> **Definition:**

Program is in SSA form if every variable is only assigned once.
Static Single Assignment (SSA)

> Each assignment to a temporary is given a unique name
  —All uses reached by that assignment are renamed
  —Compact representation
  —Useful for many kinds of compiler optimization …

```
x := 3;
x := x + 1;
x := 7;
x := x*2;
```

```
x_1 := 3;
x_2 := x_1 + 1;
x_3 := 7;
x_4 := x_3*2;
```


http://en.wikipedia.org/wiki/Static_single_assignment_form
Why Static?

> Why Static?
  — *We only look at the static program*
  — *One assignment per variable in the program*

> At runtime variables are assigned multiple times!
**Example: Sequence**

_Easy to do for sequential programs:_

**Original**

<table>
<thead>
<tr>
<th>Original</th>
<th>SSA</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a := b + c )</td>
<td>( a_1 := b_1 + c_1 )</td>
</tr>
<tr>
<td>( b := c + 1 )</td>
<td>( b_2 := c_1 + 1 )</td>
</tr>
<tr>
<td>( d := b + c )</td>
<td>( d_1 := b_2 + c_1 )</td>
</tr>
<tr>
<td>( a := a + 1 )</td>
<td>( a_2 := a_1 + 1 )</td>
</tr>
<tr>
<td>( e := a + b )</td>
<td>( e_1 := a_2 + b_2 )</td>
</tr>
</tbody>
</table>

SSA form makes clear that \( a_2 \) is not the same as \( a_1 \), so easier for analysis.
Example: Condition

Conditions: what to do on control-flow merge?

Original

if B then
  a := b
else
  a := c
end

... a ...

SSA

if B then
  a_1 := b
else
  a_2 := c
end

... a? ...

is it a1 or is it a2?
is it \(a_1\) or is it \(a_2\)?
The \( \Phi \)-Function

- \( \Phi \)-functions are always at the beginning of a basic block
- Selects between values depending on control-flow
- \( a_{k+1} := \Phi(a_1 \ldots a_k) \): the block has \( k \) preceding blocks

\( \Phi \)-functions are evaluated simultaneously within a basic block.
> SSA is normally used for control-flow graphs (CFG)

> Basic blocks are in 3-address form
Recall: Control flow graph

> A CFG models *transfer of control* in a program

— nodes are *basic blocks* (straight-line blocks of code)
— edges represent *control flow* (loops, if/else, goto …)

```
if x = y then
  S1
else
  S2
end
S3
```
SSA: a Simple Example

if B then
    a1 := 1
else
    a2 := 2
end
a3 := Φ(a1,a2)
... a3 ...

B

a1 := 1

a2 := 2

a3 := PHI(a1,a2)
... a3 ...

... a3 ...
Roadmap

- Intermediate representations
- Static Single Assignment
- **SSA generation**
  - Dominance and SSA generation
  - Applications of SSA
  - $\Phi$-congruence and SSA removal
Recall: IR

- Front end produces IR
- Optimizer transforms IR to more efficient program
- Back end transforms IR to target code
Current trend in compiler community is to use SSA as \textit{the} IR for everything in back end.
(NB: for compilers that generate machine code, not those that generate bytecode.)
Transforming to SSA

> **Problem: Performance / Memory**
  — Minimize number of inserted Φ-functions
  — Do not spend too much time

> **Many relatively complex algorithms**
  — We do not go too much into detail
  — See literature!
Minimal SSA

> Two steps:
  — Place $\Phi$-functions
  — Rename Variables

> Where to place $\Phi$-functions?

> We want minimal amount of needed $\Phi$
  — Save memory
  — Algorithms will work faster
Path Convergence Criterion

> There should be a $\Phi$ for $a$ at node $Z$ if:
> 1. There is a block $X$ containing a definition of $a$
> 2. There is a block $Y$ ($Y \neq X$) containing a definition of $a$
> 3. There is a nonempty path $P_{xz}$ of edges from $X$ to $Z$
> 4. There is a nonempty path $P_{yz}$ of edges from $Y$ to $Z$
> 5. Path $P_{xz}$ and $P_{yz}$ do not have any nodes in common other than $Z$
> 6. The node $Z$ does not appear within both $P_{xz}$ and $P_{yz}$ prior to the end
   (although it may appear in one or the other)

> *i.e.*, $Z$ is the first place where two definitions of $a$ collide
Inserted $\Phi$ is itself a definition!

while there are nodes $X,Y,Z$ satisfying conditions 1-5 and $Z$ does not contain a $\Phi$-function for a

do
  insert $\Phi$ at node $Z$.

A bit slow, other algorithms used in practice
1. block X contains a definition of \( a \)
2. block Y (Y ≠ X) contains a definition of \( a \)
3. path \( P_{xz} \) of edges from X to Z.
4. path \( P_{yz} \) of edges from Y to Z.
5. path \( P_{xz} \) and \( P_{yz} \) do not have any nodes in common other than Z
6. node Z does not appear within both \( P_{xz} \) and \( P_{yz} \) prior to the end
Roadmap

> Intermediate representations
> Static Single Assignment
> SSA generation
> **Dominance and SSA generation**
> Applications of SSA
> $\Phi$-congruence and SSA removal
Dominance Property of SSA

> Dominance: node $D$ dominates node $N$ if every path from the start node to $N$ goes through $D$.

("strictly dominates": $D \neq N")

**Dominance Property of SSA:**

1. If $x$ is used in a $\Phi$-function in block $N$, then the node defining $x$ dominates every predecessor of $N$.
2. If $x$ is used in a non-$\Phi$ statement in $N$, then the node defining $x$ dominates $N$

"Definition dominates use"

---

NB: If $x$ is used in a $\Phi$-function in $N$, then there is another path to $N$, but not to its predecessors.
Dominance is a property of basic blocks. (one Node dominates a set of nodes).
For the dominance property, "definition of $x$" thus means the basic block in which $x$ is defined.
Dominance and SSA Creation

> Dominance can be used to efficiently build SSA

> $\Phi$-Functions are placed in all basic blocks of the Dominance Frontier

—DF(D) = the set of all nodes N such that D dominates an immediate predecessor of N but does not strictly dominate N.
Dominance frontier
Node 5 dominates all nodes in the gray area.

I.e., there is no path to any of these nodes except through node 5.
Follow edges leaving the region dominated by node 5 to the region not strictly dominated by 5.

DF(5) = \{4, 5, 12, 13\}
Simple Example

DF(B1)=
DF(B2)=
DF(B3)=
DF(B4)=

Diagram:

```
B
   /\   \\
 B1 /   \ B2
       |   |
      a := 1 | a := 2

B3
   /\   \\
   B4
```
Simple Example

DF(B1)={?}
DF(B2)=
DF(B3)=
DF(B4)=

\begin{center}
\begin{tikzpicture}
\node at (-1.5,0) (B) {B};
\node at (-2.5,-2) (B2) {B2};
\node at (-1.5,-2) (B3) {B3};
\node at (0,-2) (a1) {a := 1};
\node at (1,-2) (a2) {a := 2};
\node at (0,-3) (a) {a};
\node at (1.5,0) (B1) {B1};
\node at (1.5,-2) (B4) {B4};
\draw[->] (B) -- (B2);
\draw[->] (B) -- (B3);
\draw[->] (B2) -- (a1);
\draw[->] (B3) -- (a2);
\draw[->] (a1) -- (a);
\draw[->] (a2) -- (a);
\draw[->] (a) -- (B4);
\end{tikzpicture}
\end{center}
Simple Example

DF(B1)={}  
DF(B2)=  
DF(B3)=  
DF(B4)=
Simple Example

DF(B1)=\{
DF(B2)=\{?
DF(B3)=
DF(B4)=
Simple Example

DF(B1)=\{\}
DF(B2)=(B4)
DF(B3)=
DF(B4)=

Diagram:

- B
  - B1
  - B2
    - a := 1
  - B3
    - a := 2
  - B4
    - a
Simple Example

DF(B1)={}
DF(B2)={B4}
DF(B3)={B4}
DF(B4)=

![Diagram with nodes and arrows indicating dependencies.](Image)
Simple Example

DF(B1) = {}  
DF(B2) = {B4}  
DF(B3) = {B4}  
DF(B4) = {}
Simple Example

\[
\begin{align*}
\text{DF}(B1) &= \emptyset \\
\text{DF}(B2) &= \{B4\} \\
\text{DF}(B3) &= \{B4\} \\
\text{DF}(B4) &= \emptyset
\end{align*}
\]

\[\Phi\text{-Function needed in B4 (for a)}\]
Roadmap

> Intermediate representations
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> Dominance and SSA generation
> **Applications of SSA**
> \( \Phi \)-congruence and SSA removal
Properties of SSA

> Simplifies many optimizations
  — Every variable has only one definition
  — Every use knows its definition, every definition knows its uses
  — Unrelated variables get different names

> Examples:
  — Constant propagation
  — Value numbering
  — Invariant code motion and removal
  — Strength reduction
  — Partial redundancy elimination

Constant propagation: substitute constants and evaluate constant expressions
Value numbering: number values & expressions to eliminate redundant computation
Invariant code motion and removal: move invariant code out of loops
Strength reduction: replace expensive operations by equivalent, cheaper ones (eg multiplication by addition)
Partial redundancy elimination: move common subexpressions to eliminate recomputation
SSA in the Real World

> Invented end of the 80s, a lot of research in the 90s

> Used in many modern compilers
   — ETH Oberon 2
   — LLVM
   — GNU GCC 4
   — IBM Jikes Java VM
   — Java Hotspot VM
   — Mono
   — Many more…
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Transforming out-of SSA

> Processor cannot execute $\Phi$-Function

> How do we remove it?
Naive copy placement may produce incorrect results after optimization... 

Here we simply push the assignments to a3 up to each branch. 
Sreedhar shows that the naive approach can be wrong if variables “interfere”.
**Φ-Congruence**

**Idea:** transform program so that all variables in Φ are the same:

\[
a1 = \Phi(a1,a1) \quad \Rightarrow \quad a1 = a1
\]

> Insert Copies
> Rename Variables
Φ-Congruence: Definitions

Φ-connected(x):

\[ a_3 = \Phi(a_1, a_2) \]
\[ a_5 = \Phi(a_3, a_4) \]

a1, a2, a3 are \( \Phi \)-connected
a3, a4, a5 are \( \Phi \)-connected

Φ-congruence-class:
Transitive closure of \( \Phi \)-connected(x).

a1-a5 are \( \Phi \)-congruent

x and y are connected if they are used or defined in the same \( \Phi \) instruction
The property obviously holds before optimization, since all Φ-connected variables started out as the same variable.
Liveness

A variable \( v \) is \textit{live} on edge \( e \) if there is a path through \( e \) to a use of \( v \) not passing through an assignment to \( v \).

I.e., follow paths from assignments to last use before a new assignment.

NB: \( c \) is implicitly assigned when it is defined, so is live from the start to its first use.
Interference

a and c are live at the same time: interference
Φ-Removal: Big picture

> CSSA: SSA with Φ-congruence-property.
  —directly after SSA generation
  —no interference

> TSSA: SSA without Φ-congruence-property.
  —after optimizations
  —Interference

1. Transform TSSA into CSSA (fix interference)
2. Rename Φ-variables
3. Delete Φ

CSSA = Conventional SSA
TSSA = Transformed SSA
SSA and Register Allocation

> Idea: remove $\Phi$ as late as possible

> Variables in $\Phi$-function never live at the same time!
  — *Can be stored in the same register*

> Do register allocation on SSA!

So, don’t remove $\Phi$ functions before register allocation! Keep them till end.
(Many reasons to keep SSA as IR for various phases in the back end.)
What you should know!

✎ Why do most compilers need an intermediate representation for programs?
✎ What are the key tradeoffs between structural and linear IRs?
✎ What is a “basic block”?
✎ What are common strategies for representing case statements?
✎ When a program has SSA form.
✎ What is a Φ-function.
✎ When do we place Φ-functions
✎ How to remove Φ-functions
Can you answer these questions?

✎ Why can’t a parser directly produce high quality executable code?
✎ What criteria should drive your choice of an IR?
✎ What kind of IR does JTB generate?
✎ Why can we not directly generate executable code from SSA?
✎ Why do we use 3-address code and CFG for SSA?