Thanks to Jens Palsberg and Tony Hosking for their kind permission to reuse and adapt the CS132 and CS502 lecture notes.  
http://www.cs.ucla.edu/~palsberg/  
http://www.cs.purdue.edu/homes/hosking/  

SSA lecture notes by Marcus Denker
Roadmap

> Intermediate representations
> Static Single Assignment
> SSA generation
> Dominance and SSA generation
> Applications of SSA
> $\Phi$-congruence and SSA removal

Roadmap

> Intermediate representations
> Static Single Assignment
> SSA generation
> Dominance and SSA generation
> Applications of SSA
> $\Phi$-congruence and SSA removal
Why use intermediate representations?

1. Software engineering principle
   — break compiler into manageable pieces
2. Simplifies retargeting to new host
   — isolates back end from front end
3. Simplifies support for multiple languages
   — different languages can share IR and back end
4. Enables machine-independent optimization
   — general techniques, multiple passes
IR scheme

- front end produces IR
- optimizer transforms IR to more efficient program
- back end transforms IR to target code
Kinds of IR

- Abstract syntax trees (AST)
- Linear operator form of tree (e.g., postfix notation)
- Directed acyclic graphs (DAG)
- Control flow graphs (CFG)
- Program dependence graphs (PDG)
- Static single assignment form (SSA)
- 3-address code
- Hybrid combinations
Categories of IR

> Structural
  — graphically oriented (trees, DAGs)
  — nodes and edges tend to be large
  — heavily used on source-to-source translators

> Linear
  — pseudo-code for abstract machine
  — large variation in level of abstraction
  — simple, compact data structures
  — easier to rearrange

> Hybrid
  — combination of graphs and linear code (e.g. CFGs)
  — attempt to achieve best of both worlds
Important IR properties

> Ease of generation
> Ease of manipulation
> Cost of manipulation
> Level of abstraction
> Freedom of expression (!)
> Size of typical procedure
> Original or derivative

Subtle design decisions in the IR can have far-reaching effects on the speed and effectiveness of the compiler!

⇒ Degree of exposed detail can be crucial
Abstract syntax tree

An AST is a parse tree with nodes for most non-terminals removed.

Since the program is already parsed, non-terminals needed to establish precedence and associativity can be collapsed!

A linear operator form of this tree (postfix) would be:

```
  x 2 y * –
```
A DAG is an AST with unique, shared nodes for each value.

\[
x := 2 * y + \sin(2*x) \\
z := x / 2
\]
A CFG models *transfer of control* in a program
— nodes are *basic blocks* (straight-line blocks of code)
— edges represent *control flow* (loops, if/else, goto …)

```plaintext
if x = y then
  S1
else
  S2
end
S3
```
Statements take the form: $x = y \text{ op } z$
—single operator and at most three names

Advantages:
—compact form
—names for intermediate values
## Typical 3-address codes

<table>
<thead>
<tr>
<th>assignments</th>
<th>( x = y \text{ op } z )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( x = \text{ op } y )</td>
</tr>
<tr>
<td></td>
<td>( x = y[i] )</td>
</tr>
<tr>
<td></td>
<td>( x = y )</td>
</tr>
<tr>
<td>branches</td>
<td>goto L</td>
</tr>
<tr>
<td>conditional branches</td>
<td>if ( x ) \text{ rel op } y \text{ goto } L</td>
</tr>
<tr>
<td>procedure calls</td>
<td>param ( x )</td>
</tr>
<tr>
<td></td>
<td>param ( y )</td>
</tr>
<tr>
<td></td>
<td>call ( p )</td>
</tr>
<tr>
<td>address and pointer</td>
<td>( x = &amp; y )</td>
</tr>
<tr>
<td>assignments</td>
<td>( *y = z )</td>
</tr>
</tbody>
</table>
3-address code — two variants

**Quadruples**

\[
\begin{array}{c|c|c|c}
 x - 2 \times y \\
(1) & \text{load} & t1 & y \\
(2) & \text{loadi} & t2 & 2 \\
(3) & \text{mult} & t3 & t2 & t1 \\
(4) & \text{load} & t4 & x \\
(5) & \text{sub} & t5 & t4 & t3 \\
\end{array}
\]

- simple record structure
- easy to reorder
- explicit names

**Triples**

\[
\begin{array}{c|c|c|c|c|c|c}
 x - 2 \times y \\
(1) & \text{load} & y \\
(2) & \text{loadi} & 2 \\
(3) & \text{mult} & (1) & (2) \\
(4) & \text{load} & x \\
(5) & \text{sub} & (4) & (3) \\
\end{array}
\]

- table index is implicit name
- only 3 fields
- harder to reorder
IR choices

> Other hybrids exist
  — combinations of graphs and linear codes
  — CFG with 3-address code for basic blocks

> Many variants used in practice
  — no widespread agreement
  — compilers may need several different IRs!

> Advice:
  — choose IR with right level of detail
  — keep manipulation costs in mind
Roadmap

- Intermediate representations
- **Static Single Assignment**
- SSA generation
- Dominance and SSA generation
- Applications of SSA
- $\Phi$-congruence and SSA removal
SSA: Literature

Books:
- SSA Chapter in Appel
- Chapter 8.11 Muchnik

SSA Creation:
Cytron et. al: *Efficiently computing Static Single Assignment Form and the Control Dependency Graph* (TOPLAS, Oct 1991)

Static Single Assignment Form

> Goal: simplify procedure-global optimizations

> Definition:

Program is in SSA form if every variable is only assigned once
Static Single Assignment (SSA)

Each assignment to a temporary is given a unique name
— All uses reached by that assignment are renamed
— Compact representation
— Useful for many kinds of compiler optimization …

```
x := 3;
x := x + 1;
x := 7;
x := x*2;
```

```
x1 := 3;
x2 := x1 + 1;
x3 := 7;
x4 := x3*2;
```


http://en.wikipedia.org/wiki/Static_single_assignment_form
Why *Static*?

> Why Static?
  
  — *We only look at the static program*
  
  — *One assignment per variable in the program*

> At runtime variables are assigned multiple times!
Example: Sequence

Easy to do for sequential programs:

Original

\[
\begin{align*}
a &:= b + c \\
b &:= c + 1 \\
d &:= b + c \\
a &:= a + 1 \\
e &:= a + b
\end{align*}
\]

SSA

\[
\begin{align*}
a_1 &:= b_1 + c_1 \\
b_2 &:= c_1 + 1 \\
d_1 &:= b_2 + c_1 \\
a_2 &:= a_1 + 1 \\
e_1 &:= a_2 + b_2
\end{align*}
\]
Example: Condition

Conditions: what to do on control-flow merge?

Original

```plaintext
if B then
  a := b
else
  a := c
end

... a ...
```

SSA

```plaintext
if B then
  a₁ := b
else
  a₂ := c
end

... a? ...
```
Solution: Φ-Function

Conditions: what to do on control-flow merge?

Original

\[
\text{if } B \text{ then } \\
a := b \\
\text{else } \\
a := c \\
\text{end}
\]

\[
\text{... } a \text{ ...}
\]

SSA

\[
\text{if } B \text{ then } \\
a_1 := b \\
\text{else } \\
a_2 := c \\
\text{end}
\]

\[
a_3 := \Phi(a_1, a_2) \\
\text{... } a_3 \text{ ...}
\]
The $\Phi$-Function

> $\Phi$-functions are always at the beginning of a basic block

> Selects between values depending on control-flow

> $a_{k+1} := \Phi(a_1 \ldots a_k)$: the block has $k$ preceding blocks

$\Phi$-functions are evaluated simultaneously within a basic block.
SSA and CFG

> SSA is normally used for control-flow graphs (CFG)

> Basic blocks are in 3-address form
Recall: Control flow graph

> A CFG models *transfer of control* in a program

—nodes are *basic blocks* (straight-line blocks of code)
—edges represent *control flow* (loops, if/else, goto …)

```
if x = y then
  S1
else
  S2
end
S3
```
if B then
   a1 := 1
else
   a2 := 2
end
a3 := \Phi(a1,a2)
... a3 ...

B

a1 := 1

a2 := 2

a3 := PHI(a1,a2)
... a3 ...

SSA: a Simple Example
Roadmap

- Intermediate representations
- Static Single Assignment
- **SSA generation**
- Dominance and SSA generation
- Applications of SSA
- $\Phi$-congruence and SSA removal
Recall: IR

- front end produces IR
- optimizer transforms IR to more efficient program
- back end transforms IR to target code
SSA as IR
Transforming to SSA

> **Problem: Performance / Memory**
  — Minimize number of inserted $\Phi$-functions
  — Do not spend too much time

> **Many relatively complex algorithms**
  — We do not go too much into detail
  — See literature!
Minimal SSA

> Two steps:
  — Place $\Phi$-functions
  — Rename Variables

> Where to place $\Phi$-functions?

> We want minimal amount of needed $\Phi$
  — *Save memory*
  — *Algorithms will work faster*
There should be a $\Phi$ for $a$ at node $Z$ if:

1. There is a block $X$ containing a definition of $a$
2. There is a block $Y$ ($Y \neq X$) containing a definition of $a$
3. There is a nonempty path $P_{xz}$ of edges from $X$ to $Z$
4. There is a nonempty path $P_{yz}$ of edges from $Y$ to $Z$
5. Path $P_{xz}$ and $P_{yz}$ do not have any nodes in common other than $Z$
6. The node $Z$ does not appear within both $P_{xz}$ and $P_{yz}$ prior to the end (although it may appear in one or the other)

I.e., $Z$ is the first place where two definitions of $a$ collide
Iterated Path-Convergence

> Inserted $\Phi$ is itself a definition!

```plaintext
while there are nodes $X,Y,Z$ satisfying conditions 1–5 and $Z$ does not contain a $\Phi$-function for a
do
  insert $\Phi$ at node $Z$.
```

A bit slow, other algorithms used in practice
Example (Simple)

1. block X contains a definition of a
2. block Y (Y ≠ X) contains a definition of a
3. path \( P_{xz} \) of edges from X to Z.
4. path \( P_{yz} \) of edges from Y to Z.
5. path \( P_{xz} \) and \( P_{yz} \) do not have any nodes in common other than Z
6. node Z does not appear within both \( P_{xz} \) and \( P_{yz} \) prior to the end
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Dominance Property of SSA

> Dominance: node $D$ dominates node $N$ if every path from the start node to $N$ goes through $D$.

(“strictly dominates”: $D \neq N$)

Dominance Property of SSA:

1. If $x$ is used in a $\Phi$-function in block $N$, then the node defining $x$ dominates every predecessor of $N$.
2. If $x$ is used in a non-$\Phi$ statement in $N$, then the node defining $x$ dominates $N$

“Definition dominates use”
Dominance and SSA Creation

> Dominance can be used to efficiently build SSA

> Φ-Functions are placed in all basic blocks of the *Dominance Frontier*

—DF(D) = the set of all nodes N such that D dominates an immediate predecessor of N but does not strictly dominate N.
Dominance frontier
Node 5 dominates all nodes in the gray area
Follow edges leaving the region dominated by node 5 to the region not strictly dominated by 5.

DF(5) = \{4, 5, 12, 13\}
Simple Example

DF(B1)=
DF(B2)=
DF(B3)=
DF(B4)=

Diagram:

B → B1 → B2 → a := 1 → a
B → B3 → a := 2 → a
B → B4 → a
Simple Example

DF(B1) = {?}
DF(B2) =
DF(B3) =
DF(B4) =
Simple Example

\[ \text{DF(B1)} = \{ \} \]
\[ \text{DF(B2)} = \]
\[ \text{DF(B3)} = \]
\[ \text{DF(B4)} = \]
Simple Example

DF(B1) = \{\}
DF(B2) = \{?\}
DF(B3) = 
DF(B4) =
Simple Example

DF(B1) = {} 
DF(B2) = {B4} 
DF(B3) = 
DF(B4) =
Simple Example

DF(B1) = {}
DF(B2) = {B4}
DF(B3) = {B4}
DF(B4) =
Simple Example

DF(B1)={}
DF(B2)={B4}
DF(B3)={B4}
DF(B4)={}

a := 1

B2

B1

B

B3

a := 2

B3

B1

B

B4

a
Simple Example

\[ \Phi - \text{Function needed in } B4 \text{ (for } a) \]

DF(B1)={}
DF(B2)={B4}
DF(B3)={B4}
DF(B4)={}

\[
\begin{align*}
B & \to B1 \\
B2 & \to a := 1 \\
& \quad \rightarrow \\
& \quad \rightarrow \\
B3 & \to a := 2 \\
& \quad \rightarrow \\
& \quad \rightarrow \\
B4 & \to a
\end{align*}
\]
Roadmap

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Properties of SSA

> Simplifies many optimizations
  — Every variable has only one definition
  — Every use knows its definition, every definition knows its uses
  — Unrelated variables get different names

> Examples:
  — Constant propagation
  — Value numbering
  — Invariant code motion and removal
  — Strength reduction
  — Partial redundancy elimination
SSA in the Real World

> Invented end of the 80s, a lot of research in the 90s

> Used in many modern compilers
  — *ETH Oberon 2*
  — *LLVM*
  — *GNU GCC 4*
  — *IBM Jikes Java VM*
  — *Java Hotspot VM*
  — *Mono*
  — *Many more…*
Roadmap

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Transforming out-of SSA

> Processor cannot execute Φ-Function

> How do we remove it?
Naive copy placement may produce incorrect results after optimization …
**Φ-Congruence**

**Idea:** transform program so that all variables in Φ are the same:

\[ a_1 = Φ(a_1, a_1) \quad \Rightarrow \quad a_1 = a_1 \]

> Insert Copies
> Rename Variables
**Φ-Congruence: Definitions**

**Φ-connected(x):**

\[ a_3 = \Phi(a_1, a_2) \]
\[ a_5 = \Phi(a_3, a_4) \]

\[ a_1, a_2, a_3 \] are \( \Phi \)-connected
\[ a_3, a_4, a_5 \] are \( \Phi \)-connected

**Φ-congruence-class:**

Transitive closure of \( \Phi \)-connected\((x)\).

\[ a_1-a_5 \] are \( \Phi \)-congruent
Φ-Congruence Property

Φ-congruence property:

All variables of the same congruence class can be replaced by one representative variable without changing the semantics.

SSA without optimizations has Φ-congruence property

Variables of the congruence class never live at the same time (by construction)
Liveness

A variable \( v \) is *live* on edge \( e \) if there is a path through \( e \) to a use of \( v \) not passing through an assignment to \( v \).

\[\begin{array}{c}
\text{a:= 0} \\
b := a + 1 \\
c := c + b \\
a := b \times 2 \\
a < N \\
\text{return c}
\end{array}\]

\[\begin{array}{c}
\text{a:= 0} \\
b := a + 1 \\
c := c + b \\
a := b \times 2 \\
a < N \\
\text{return c}
\end{array}\]

\[\begin{array}{c}
\text{a:= 0} \\
b := a + 1 \\
c := c + b \\
a := b \times 2 \\
a < N \\
\text{return c}
\end{array}\]

\[a \text{ and } b \text{ are never live on the same edges, so two registers suffice to hold } a, b \text{ and } c\]
Interference

\[ a := 0 \]
\[ b := a + 1 \]
\[ c := c + b \]
\[ a := b \times 2 \]
\[ a < N \]
\[ \text{return } c \]

\[ a := 0 \]
\[ b := a + 1 \]
\[ c := c + b \]
\[ a := b \times 2 \]
\[ a < N \]
\[ \text{return } c \]

\[ a := 0 \]
\[ b := a + 1 \]
\[ c := c + b \]
\[ a := b \times 2 \]
\[ a < N \]
\[ \text{return } c \]

\textit{a and c are live at the same time: interference}
Φ-Removal: Big picture

> CSSA: SSA with Φ-congruence-property.
  — *directly after SSA generation*
  — *no interference*

> TSSA: SSA without Φ-congruence-property.
  — *after optimizations*
  — *Interference*

1. Transform TSSA into CSSA (fix interference)
2. Rename Φ-variables
3. Delete Φ
Idea: remove $\Phi$ as late as possible

Variables in $\Phi$-function never live at the same time!

—Can be stored in the same register

Do register allocation on SSA!
What you should know!

- Why do most compilers need an intermediate representation for programs?
- What are the key tradeoffs between structural and linear IRs?
- What is a “basic block”?
- What are common strategies for representing case statements?
- When a program has SSA form.
- What is a $\Phi$-function.
- When do we place $\Phi$-functions
- How to remove $\Phi$-functions
Can you answer these questions?

- Why can’t a parser directly produce high quality executable code?
- What criteria should drive your choice of an IR?
- What kind of IR does JTB generate?
- Why can we not directly generate executable code from SSA?
- Why do we use 3-address code and CFG for SSA?
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