

Solution Lambda Calculus

Instructions:

Solutions of the exercises are to be delivered before Wednesday, the 11th of April at 10:15AM.

Solutions should be placed in a separate folder with the name "Assignment05".

Please submit answers to all the exercises in **one** text file.

Exercise 1 (2 points)

Consider the following λ -expressions. Indicate which occurrences of variables are bound and which ones are free in the expressions.

1. $(\lambda x . x) y (\lambda y . y x) x$

2. $((\lambda x . \lambda y . \lambda z . x y z) (\lambda x . y x) y) (\lambda x . z x)$

Answer:

b = bound

f = free

ex. 1.1.

$$\begin{array}{ccccccc} (\lambda x . x) & y & (\lambda y . y x) & x \\ | & | & | & | & | \\ b & f & b & f & f \end{array}$$

ex. 1.2.

$$\begin{array}{ccccccc} ((\lambda x . \lambda y . \lambda z . x y z) & (\lambda x . y x) & y) & (\lambda x . z x) \\ | & | & | & | & | \\ b & b & b & f & b & f & f & b \end{array}$$

Exercise 2 (2 points)

Define boolean functions `and` and `or` in Lambda Calculus and show that `True and False = False` and `True or False = True` based on the definitions of `True` and `False` functions from the lecture hours.

Answer:

$and \equiv \lambda x y . x y x$

// if the first argument is *False*, the function should return *False*, i.e., the first argument;

// if the first argument is *True*, the function should return the second argument

$or \equiv \lambda x y . x x y$

// if the first argument is *True*, the function should return *True*, i.e., the first argument;

// if the first argument is *False*, the function should return the second argument

True and False = False

$(\lambda x y . x y x)(\lambda x y . x)(\lambda x y . y) =$

$(\lambda x y . x)(\lambda x y . y)(\lambda x y . x) =$

$(\lambda x y . y) \equiv False$

True or False = True

$(\lambda x y . x x y)(\lambda x y . x)(\lambda x y . y) =$

$(\lambda x y . x)(\lambda x y . x)(\lambda x y . y) =$

$(\lambda x y . x) \equiv True$

Exercise 3 (2 points)

Reduce the following λ -expressions to their normal form where possible.

1. $(\lambda x . (\lambda z . z y) x) (\lambda x . x)$

2. $(\lambda x . x x y) (\lambda x . x x y)$

Answer:

a. $(\lambda x . (\lambda z . z y) x)(\lambda x . x) = /* \beta \text{ reduction} */$

$(\lambda z . z y)(\lambda x . x) = /* \beta \text{ reduction} */$

$(\lambda x . x) y = /* \beta \text{ reduction} */$

y

b. $(\lambda x . x x y)(\lambda x . x x y) = /* \beta \text{ reduction} */$

$(\lambda x . x x y) (\lambda x . x x y) y = /* \beta \text{ reduction} */$

$(\lambda x . x x y) (\lambda x . x x y) y y = /* \beta \text{ reduction} */$

... */* no normal form */*