Graph

A data structure consisting of a set of vertices (V) connected by a set of edges (E) that can be used to model relationships among the objects in a collection.

\[ G = (V, E) \]
\[ E = \{ (i, j) | i, j \in V \} \]

Graphs are also referred to as networks, vertices as nodes, and edges as links.

What is it for

You can imagine any transportation or transmission system.
- Blood vessels, nerves, communication systems (e.g. network)

For instance, you see unique search results in Facebook based on:
- Your connections to people, places, things.
- What you're able to see on Facebook, including what your friends share with you.
- Your friends, and interests, which affect the order of your results.
- People’s privacy settings. For example, if you search “photo Paris,” you may see photos your friends took and shared with you first.

Graph properties

- Undirected graph
- Directed graph
- Simple graph
- Complete graph
Graph properties (continue)

Complete graph $V'$, $V$.

Graph Implementation

```
public class Graph {
    List<Node> nodes;
}
```

```
public class Edge {
    public Node start;
    public Node end;
    public double weight;
}
```

```
public class Node {
    public String name;
    public List<Edge> connections;
}
```

Array representation of a graph

```
A B C D E
\[ 1 1 0 0 0 \\
2 1 1 1 0 \\
3 0 1 1 0 \\
4 0 0 1 1 \\
5 0 0 0 1 \\
\]
```

```
G = \[
\begin{pmatrix}
1 & 2 & 3 \\
2 & 1 & 0 \\
3 & 0 & 1 \\
4 & 1 & 3 \\
5 & 0 & 0 \\
\end{pmatrix}
\]
```
Degree of a vertex in a matrix

\[ G = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix} \]

\[ \text{Degree}(V_i) = \sum_j G_{i,j} + \sum_j G_{j,i} \]

N.B. If a graph is undirected then its corresponding matrix is symmetric and for every vertex: \( \text{Degree}(V) = \text{in-degree} = \text{out-degree} \)

Exercise

Given two vertices \( V_i \) and \( V_j \) in a graph, compute the shortest path between these two vertices with the help of the corresponding matrix of the graph.