

*i*STOA: Artefacts for mathematical interactive learning exercises

Hilaire Fernandes^{1,3}

Thibault Carron²

Stéphane Ducasse¹

¹ INRIA-Futurs Lille, CNRS-UMR 8022, LIFL - Université de Lille

² Université de Savoie, SYSCOM labs

SCÉRÉN - CRDP Aquitaine

hilaire.fernandes@crdp-aquitaine.fr, stephane.ducasse@inria.fr, thibault.carron@univ-savoie.fr

Abstract

In primary schools, mathematics teachers use support tools to introduce new concepts. The objective of these tools is to reinforce a mental representation of the newly introduced concept. Tools can be physical objects or paper-pen based. We call these tools artefacts. In computer assisted environments, such artefacts are not always clearly present, those environments focus on the nature of the exercises (drills, quiz). To realise environments in closer relation to classroom teaching, we propose to analyse and categorise such artefacts: we used pedagogical literature and we extracted artefacts used in teaching multiplication. We present our infrastructure and a list of artefacts in the multiplication realm.

1. Introduction

When a teacher prepares a learning activity, s/he introduces special tools acting as *auxiliaries* between the learner, the concept and his/her activity[15]. These auxiliaries help the learner to construct a mental representation of the concept.

We name these tools *artefacts*. Such tools can be physical objects (tokens of different colours, dice, ...) or paper-pen environment (to draw grid, to draw a series of object,...). The artefact is most of the time a preexisting object instrumented in a unique manner to support a learning process. It's an intermediate between the learnt concept and the internal representation of the learner. Thanks to it, the learner internalises the concept step by step. Most of the time, it is the first object the learner interacts with. Teachers use such artefacts in pedagogical situations and exercises they prepare for their students. Such pedagogical situations are contextualised in step by step progressive learning processes or *pedagogical scaffolding*[7].

The idea of automated or self learning is a very ancient dream of the humanity. Automates and computer science revitalised this idea through different forms: programmed education in the form of drilling systems such as the Sidney Pressey one's [19] were automatic quiz machine, intelligent tutors[4] propose intelligent systems capable of guiding the learner when experiencing difficulties. Micro-world[6] environments are models of a specific world where the learner can manipulate the world objects according to internal rules of the world (logo, interactive geometry[11]). And in closer relation with unformal learning, Douglas Engelbart very brightly considered 40 years ago hypertext and collaborative environment as augmented intelligence tool for the future[10]. As a proof of this concept his team invented in the sixties the mouse, the windows, the hypertext and the collaborative tools we know today.

Today, interactive learning environment[20] focus on interactivity and communication, eventually the e-learning direction introduces contradicting challenges between richer and interactive environments and the restriction imposed by the Internet medium.

Developing richer interactive environments is expensive in term of development resources. For each set of learning concept, new specific set of objects may need to be designed. Such programmed objects can only be developed by computer science specialists. However the core of the pedagogical concepts and the know-how about teaching come from teachers. There is a possible tension point where collaboration is needed between the teachers and computer science specialists.

Authoring systems as WIMS [12], HotPotatoes [2], Jcllic[3] allow teachers to write pedagogical exercises. These tools enable teachers to get to the contents creation process. However the teachers can only use pre-programmed elements. These elements focus essentially on the exercise nature: multiple-choice questions, short-answers, jumbled-sentences, crosswords, matching/ordering and gap-fill exercises. Therefore, the focus is

on the exercises as presented to the learner and not on its constituting part. In particular the reflexion related to the use of artefacts and tools is poor.

With computer assisted environment we are facing a contradicting challenge: on the one hand we have authoring system but it results in systems not as flexible as it should be, on the other hand, we can design systems with richer interactive contents but it requires programming skills most teachers don't possess. In all cases, the know-how about learning and teaching comes from teachers and pedagogical documentation and it must be used in any design of a learning environment.

An intermediate solution between authoring systems and full-programmed contents is to modularize as much as possible the various elements used in traditional classroom activities, to be able to reuse them in various computerised situations. We have explored this hypothesis in *iSTOA*¹, the interactive learning environment for primary school we are developing. We propose a modularisation of the different constituting parts of a learning activity. Such modularisation helps to: identify and design artefacts in a reusable way; design reusable pedagogical exercises based on artefacts; design pedagogical scaffolding based on exercises. This modularisation is the ground for an authoring system where teachers could design pedagogical scaffolding, artefacts based.

In this paper, after discussing instrumented activities, we present the overall *iSTOA* model. Next we expose examples of artefacts extracted from the reference pedagogical literature in mathematics and the implementation of these artefacts. Then we relate feedback received from an experiment in a primary school and we conclude and open perspectives with authoring system.

2. Artefacts in pedagogy

To explain and to understand what an artefact could be, we focused first on mathematics. When the multiplication concept is introduced in primary school, the teacher can use tokens to represent the product of two numbers. Thus, the product 2×3 is represented as a rectangular series of tokens in 2 rows and 3 columns. Alternatively, he can use paper and pen to draw a grid with 2 rows and 3 columns. What we call artefact is therefore a tool used to give the learner physical, touchable, kinetic, viewable sensation of an intellectual concept.

According to the Vygotsky theory of activity, the learner always operates on an activity with an intermediate auxiliary, a tool. This auxiliary may impose a representation, more or less internalized by the learner. This representation depends on the learner and the domain[5].

¹In the building of ancient Greece, a stoa is a covered walkways commonly used as public place for ideas exchange and commerce

For Béguin [9], the tool is at first an artefact instrumented in a unique manner by each learner. Two different persons, according to their personal experiences, can use a given tool differently: the artefact is instrumented in two different ways. The way an artefact is instrumented depends on the learner's knowledge, the activity and the environment.

Related to our previous example, we come to the conclusion that the teacher is proposing to his/her students a conscious instrumentation of an artefact. The teacher decides to instrument an artefact to impose to the learner a mental representation and to achieve an intended pedagogical realisation.

In our example, the artefact (token, grid) can be embedded in two related exercises: a first exercise asking the learner to represent the product 2×3 as a rectangular series of tokens. Then in a second exercise, depending on the learner's answer to the first one (let's say a 2 rows and 3 columns series), a next question concerns another rectangular series of token (here, 3 rows and 2 columns series). This two-step pedagogical scaffolding objective is to show a product can have several rectangular representations.

As aforesaid, an artefact is usable in different exercises – with different instrumentations. For computerised artefact, we also want different instrumentation. This means reusable and parametrised artefacts. It is an important point to keep in mind when developing computerised artefacts.

An instrumented artefact – a tool – is closely tied to a concept. Our *tokens used in rectangular series* are specific to the introduction of the multiplication concept and can't be used for a different concept. However the token artefact itself is usable to approach different concept as the addition or subtraction concepts.

Therefore the artefact appears to us as the smallest computerised interactive unit. The artefact itself is then instrumented as an unique tool, for a specific intended pedagogical result. The tool itself is embedded in an interactive exercise. The exercise is characterised by the learner's prerequisite knowledge and pedagogical achievements, it is part of a pedagogical scaffolding. In *iSTOA*, the interactive units follow this pattern, the next section exposes its different parts and how they are related.

3. The *iSTOA* model

In such pedagogical environments and especially those we want to develop, several pedagogical elements are linked to the artefact: the concerned concepts, the exercises helping to acquire one of these concepts, the scaffolding of exercises... Each of these elements will be described more precisely in the unit in which they have been put together. As a matter of fact, our model is based on four units:

- The knowledge network unit describing a whole net-

environment [13, 16] and the Morphic[17] system to implement the artefacts. In Squeak, a Morph is a composite direct manipulation user interface element. The *iSTOA* learning unit is designed to be smoothly integrated in the Morphic system, and to benefit from the whole dynamic offered by Smalltalk.

3.3. The Recording Unit

The aim of this unit is to record the learner produced exercises and his/her learning status. The system uses these records to adapt and to propose adequate exercises. This unit is also a mouse and keyboard event recorder to replay cinematically the learner's actions. It is an important tools for the teacher to review how the learner conducted his/her exercise. Very often interesting information about the learner's activity are only visible while the learner is doing the exercise, not when the exercise is done.

This unit is implemented using an object oriented database. Thanks to the oriented-object nature of the database, we record very high-level object (class instances) independently of the definitions of these objects. This is very important for our environment designed with an important variety of artefacts and exercise objects. We record mainly the knowledge network and the learner achievements.

3.4. The Teacher Console Unit

The teacher console gives access to the learner's records. From there the teacher can review the learner's knowledge model, analyse the learner's achievements and replay the learner's work for each exercise.

It is now well known that in order to take advantage of these new learning management system, it is important to exploit the observation abilities and thus offer a mean for observing the progression of the learning session[18].

We chose here to record all the actions performed by the learners. From this console unit, the teacher can review the learner's knowledge model, analyse the learner's achievements and replay the learner's work for each exercise. Moreover, such tools allow the teacher to detect accurately a misunderstanding error and eventually to propose an adapted correction: remediation (a specific set of new exercises, a more detailed explanation, a part of a lesson).

Such records might also be provided to the learner to observe his/her own progression or even to show the best solution found amongst the solutions of all the learners of the classroom.

4. Models and artefacts extraction

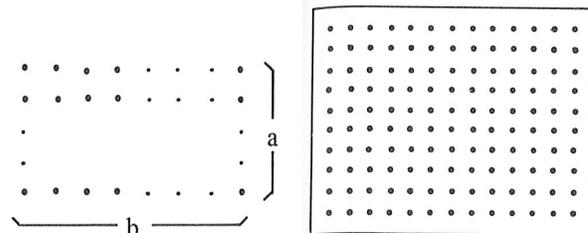
In our problem, we are concerned with the identification of the artefacts to be computerised. It is obvious, artefact definitions depend on the teaching domain. We have concentrated our effort on the introduction of the multiplication concept up to the multiplication technique of two integer numbers. This domain is interesting because it covers several artefacts. Our process is therefore a computerisation of artefacts used in the classroom.

Assisted with a mathematics teacher, we have analysed a large body of pedagogical documentation covering this subject. We focused on two types of resources: teacher pedagogical resources from the ERML work group at INRP [1] and student textbook [21].

In the following demonstration, we show the important artefacts used while teaching multiplication concept then their computerised counterparts for the *iSTOA* environment.

Rectangle of dots. The product $a \times b$ is introduced with a rectangle of dots.

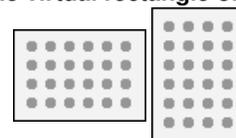
Figure 3. In the literature: rectangle of dots



A large variety of exercises can be built with this artefact. A rectangle of dots 8×9 can be represented as $8 + 8 + \dots + 8$, 8 lines of 9 dots, 9 columns of 8 dots, etc.

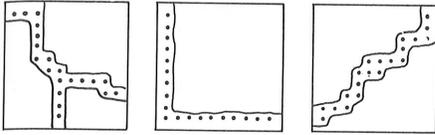
We produced a Morph artefact to represent this object. Our artefact is parametrised, so rectangle of dots with different size are easy to construct. This also lets us choose the orientation of the produced virtual artefact. In figure 4, the same rectangle of dots with different orientation represents the same product 4×6 .

Figure 4. The virtual rectangle of dots artefact



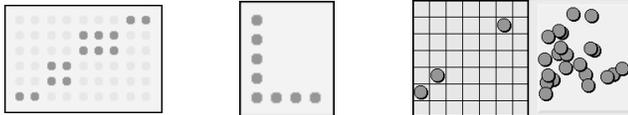
Partial dot rectangle. In the literature, partial dot rectangles are used to understand that the number of dots in a rectangle depends on the number of columns and lines (figure 5).

Figure 5. In the literature: partial dot rectangles



To make the understanding of partial dot rectangle easier, our virtual artefact comes with an option to show the hidden dots partially. This option came after an experimentation in a classroom. We show two artefacts with hidden dots on the left of figure 6.

Figure 6. Virtual partial dot rectangles (left) and interactive grid with tokens (right)



Dynamic grid of dots. The previous artefact is used in conjunction with real grid and tokens to reconstruct the initial complete rectangle of dots. Therefore we designed an interactive grid with tokens. On the right of figure 6 is an interactive grid with a basket of tokens. If necessary, the learner uses it to reconstruct a complete rectangle.

Series of series. To understand the writing $a \times b$ as a general way to express series of item series, we associate this writing to different type of series, not only to rectangle of tokens. Thus, the literature proposes different visual representations as shown in figure 7 (left).

At the right of figure 7, our artefact shows a representation of 4×6 as a series of 6-items. The tokens can be moved around so learner with difficulties can reconstruct a rectangle of tokens.

Visual decomposition of a product. To calculate the product of two numbers, number decomposition is a key point. A rectangular grid cut in two pieces is used as a support to decompose a product like 6×7 as $6 \times 5 + 6 \times 2$. The left of figure 8 shows an artefact used in the literature. On the right we see one example we designed for *i*STOA.

Figure 7. Series, in the literature and in computerised form

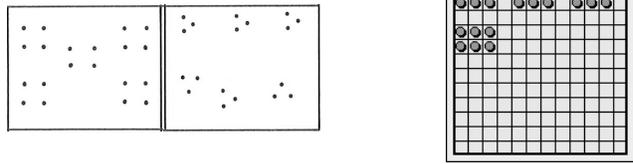
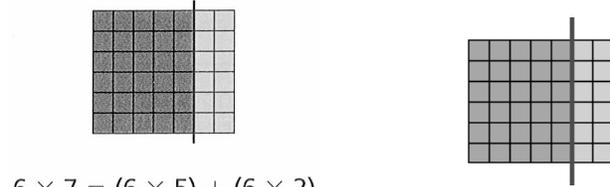


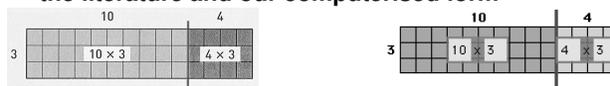
Figure 8. Decomposition in sub parts, a grid divided in two parts



$$6 \times 7 = (6 \times 5) + (6 \times 2)$$

Visual decomposition of product with text fields. Next the decomposition is used to calculate a product of two numbers: more elaborated artefacts, still using rectangular grid but with additional places to write down the decomposition, are used. To calculate 14×3 , the learner decomposes the initial product as the easier products 10×3 and 4×3 . Therefore the artefact comes with additional text zones to input the intermediate products.

Figure 9. Product calculus with decomposition, in the literature and our computerised form



In figure 9, the virtual artefact comes with input fields to enter the numbers (here already entered).

Expanded vertical writing of a product. To abstract step by step the writing of a product, an expanded form is introduced (figure 10). It is just a vertical sum of the simplified products elaborated with the previous artefact in figure 9.

Simplified vertical writing of a product. Finally, a simplified form is introduced (figure 11). This form uses intensively the mental image the learner elaborated step by step with the previous artefacts.

In the previous examples, one number of the product – 3 – was below ten. The same decomposition technique is used

Figure 10. Product calculus, expanded form, in the literature and our computerised form

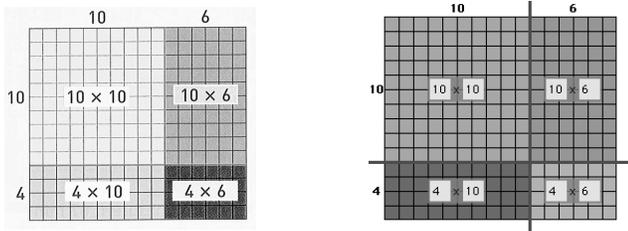


Figure 11. Product calculus, simplified form, in the literature and our computerised form



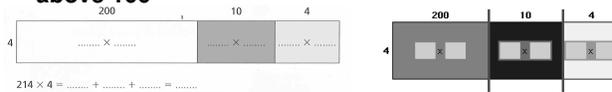
with both numbers above ten. The rectangular grid artefact just needs to be extended to cover a product decomposition as a sum of four simpler products (figure 12). Here the product 16×14 is decomposed to simpler products 10×10 , 10×6 , 4×10 and 4×6 .

Figure 12. Product calculus, both numbers above 10



Visual decomposition of product with numbers above one hundred. With numbers above one hundred, it becomes difficult to use rectangular grid proportional to the number. Nevertheless, it is a good opportunity to abstract a little more the artefact: in the literature the grid structure is dropped and only rectangular areas are used. These rectangular areas are not proportional anymore to the number. It results in artefact looking like the one in figure 13. Here the product 214×4 is decomposed on simpler products 200×4 , 10×4 and 4×4 .

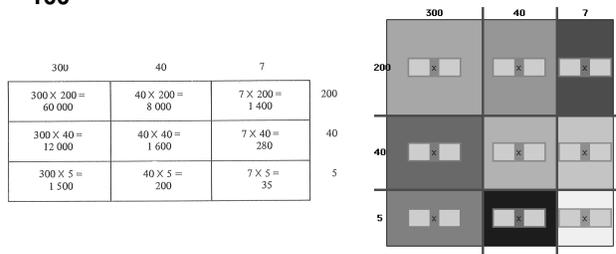
Figure 13. Product calculus, with one number above 100



With both numbers above one hundred, a more complex decomposition is necessary. Still, the artefact helps to foster a mental representation of the product through decomposition and recombination of smaller and easier products.

In the example of figure 14, the product 347×245 is calculated as the sum of the simple products 300×200 , 40×200 , 7×200 , 300×40 , 40×40 , 7×40 , 300×5 , 40×5 and 7×5 . Next an expanded and simplified vertical forms sum are used (not shown there).

Figure 14. Product calculus, both numbers above 100



So far we have discussed artefacts covering the following domains:

- Static rectangular series of dots, with or without hidden part.
- Dynamic grid of dots. The series can be prearranged in rectangular form, series of series, at random. The grid is dynamic because it can be rearranged by the learner.
- Dynamic grid of dots with a basket of dots. It is a grid the learner can rearrange dots and can use a basket to add or to remove dots from the grid.
- interactive representation of units, group of tens, hundreds with basket and targets (exposed in next section).
- Visual decompositions of product with numbers with or without text fields to write the decomposition.
- Expanded and simplified form to write a product vertically.

Based on these artefacts, we can now designed exercises and pedagogical scaffolding.

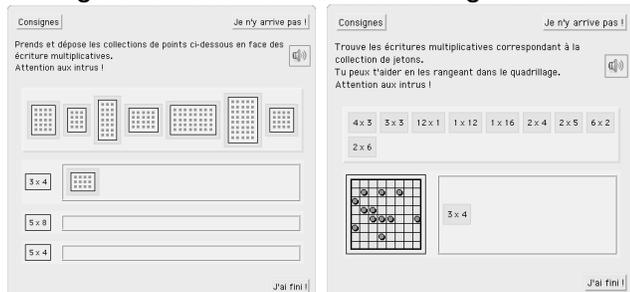
5. Examples based in a real experiment

In the following, we introduce implantation of artefacts in exercises. These exercises are grouped in 6 pedagogical scaffolding. Each scaffolding contains from 4 to 8 exercises. We do not present the scaffolding but only a few examples. These pedagogical scaffolding were used to conduct a first experiment with primary school students. We relate there the user's feedback from this experimentation.

5.1. Preparation of the exercises

The artefacts we presented so far, cover part of the multiplication concept. We combine and reuse these artefacts to produce interactive exercises. For example, we have a model for association exercise: a sticker must be correctly associated with targets. This association exercise is used to build quizzes. Both stickers and targets can be static or interactive artefacts. Figure 15 on the left illustrates an association exercise with rectangle of tokens artefacts. The learner drags an artefact from the top and he drops it next to the appropriate target – here a product representation of a number.

Figure 15. Association exercise using artefacts



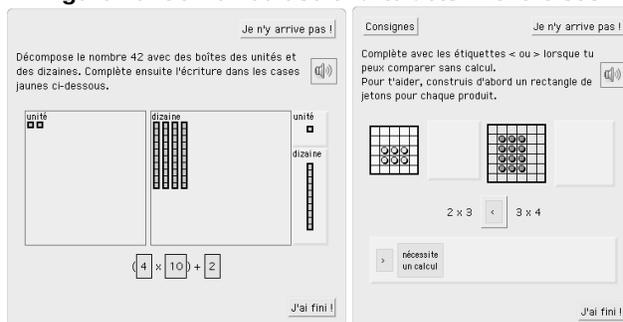
More interestingly, both the stickers and the target can be interactive artefacts. Figure 15 on the right demonstrates an exercise where the target is an interactive grid of tokens. With the help of this target, the learner seeks rectangular representations to found out the matching products.

However, a lot of exercises need a special model to define the exercise logic. In this case, an exercise is constituted of two classes: a model to handle the logic and a view – a Morph – the graphical representation. When a new exercise is designed, the model and the view are specialised. In the view, several artefacts can be used, the model handles the logic between them, this is the case in the examples we present below.

The decomposition of a number in base 10 is important to understand the decimal numeration. Later, it is used in the multiplication technique. In a classroom, tokens representing unit, group of ten, hundreds are used to decompose

numbers *with hands*. To achieve this result in *iSTOA*, we combine several artefacts in one exercise: in figure 16 on the left, artefacts representing units, group of tens, baskets are combined. The learner drags and drops unit and groups of ten in special baskets. Later he/she can give a formal decomposition with product and addition.

Figure 16. Combined use of artefacts in exercises



Just like in real situations, our artefacts can be combined when needed. In figure 16, the exercise on the right shows another example of artefacts combination. There, two grids and tokens baskets artefacts are combined to compare two products: the bigger the rectangle is, the greater the product is. A drag and drop sticker is then used to provide the answer.

5.2. Feedback concerning the experiment

Very early, we experimented in a primary school the usability of the artefacts we described. As it came very early during the development phase, not all the artefacts described in this paper were used. The students of this class were 8 years old. The teacher uses the environment with small groups of students. We got the following feedback from the teacher regarding the usability of artefacts:

- The partial dot rectangle (figure 6) was first designed without the semi-visible dots (picture on the left). So dots were visible or not (picture in the middle). This was not very easy at first for student to understand that the rectangle was semi-visible. The teacher suggested to print hidden dot semi-visible. Therefore we have extended the artefact with an option to print the hidden dot in a close to subliminal colour.
- When artefacts are used together in the same exercise, the affordance of the artefacts should match when there is a conceptual connexion between the two. In some exercises we use a partial dot rectangle and an interactive grid with tokens (figure 6). These two artefacts were designed at different moment, and we decided

later we could use them together in some exercises. So at first the tokens were green squares and the dot in the rectangle were black discs. The experimentation shows it was very important that colours and shapes should match. Otherwise, the students may not make the mental link between the two artefacts. So we added more parameters to these artefacts.

This modest feedback shows us the importance of early experimentation with learners, so that mislead artefacts can be adjusted. New ones will follow with an on-line version to get more feedback and to allow self practise.

6. Conclusion

To elaborate meaningful pedagogical scaffolding we analysed the pedagogical literature from the macro-level down to the micro-level: identification of the proposed pedagogical scaffolding and its sequencing in exercises. In turn each exercise is analysed and the used artefacts are extracted. Then the computerisation follows the reverse path: the extracted artefacts, exercises, scaffolding are computerised in this order. We have used this pattern for mathematical contents, in the future we want to use it for different domains.

These tools – artefacts – are important to support the initial learning phase, they provide physical support to foster mental image. Both the artefacts and the exercises are parametrised in different pedagogical scaffolding.

We use a three layers system with artefact, exercise and scaffolding objects. These objects can be combined independently to design more contents. The exercises are constituted of one model object – to design its logic – and one view object. The view is an aggregation of artefacts and other GUI Morph widget. We have extracted the pedagogical knowledge from specialised literature, however such knowledge can come directly from teachers. In the future, we want to open up the design of exercises to teachers. The Easy Morphic GUI [8] – EMG – is a framework to construct graphically Morph view and to link them with appropriate models. With a combined use of EMG and the interactive nature of Smalltalk programming, we could provide teachers a high level framework to design richer exercises, artefacts based.

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