

Solution Lambda Calculus

- Exercises are given every week on the PL page of the SCG website (<http://scg.unibe.ch/teaching/pl>)
- Solutions to each assignment must be sent to **mohammadreza.hazhirpasand@inf.unibe.ch**
- The solutions of the assignments are to be delivered before every Thursday at 11 PM. Solutions handed in later than the specified time will not be accepted. In case of serious reasons send an e-mail to **mohammadreza.hazhirpasand@inf.unibe.ch**

Exercise (6 points)

1. Consider the following λ -expressions. Indicate which occurrences of variables are bound and which ones are free in the expressions. (3 pts)

- (a) $(\lambda x . x) y (\lambda y . y x) x$
 (b) $((\lambda x . \lambda y . \lambda z . x y z) (\lambda x . y x) y) (\lambda x . z x)$
 (c) $\lambda y . (\lambda x . z(x(\lambda x . y(z)))) (\lambda z . y(x(z)))$

Answer:

b = bound
f = free

ex. a

$$\begin{array}{ccccccc} (\lambda x . x) y (\lambda y . y x) x \\ | \quad | \quad \quad | \quad | \quad | \\ b \quad f \quad \quad b \quad f \quad f \end{array}$$

ex. b

$$\begin{array}{ccccccc} ((\lambda x . \lambda y . \lambda z . x y z) (\lambda x . y x) y) (\lambda x . z x) \\ \quad \quad \quad | \quad | \quad | \quad \quad | \quad | \quad | \quad \quad | \quad | \\ \quad \quad \quad b \quad b \quad b \quad \quad f \quad b \quad f \quad \quad f \quad b \end{array}$$

ex. c)

$$\begin{array}{ccccccc} \lambda y . (\lambda x . z(x(\lambda x . y(z)))) (\lambda z . y(x(z))) \\ \quad \quad | \quad | \quad \quad | \quad | \quad \quad | \quad | \quad | \\ \quad \quad f \quad b \quad \quad b \quad f \quad \quad b \quad f \quad b \end{array}$$

2. If possible, reduce the following λ -expressions to their normal form. (3 pts)

- (a) $(\lambda x . (\lambda z . z y) x) (\lambda x . x)$
 (b) $(\lambda x . x x y) (\lambda x . x x y)$
 (c) $P \equiv (\lambda x . x (x y)) I$ where $I \equiv \lambda u . u$

Answer:

a. $(\lambda x. (\lambda z. z y) x)(\lambda x. x) = /* \beta \text{ reduction */}$
 $(\lambda z. z y)(\lambda x. x) = /* \beta \text{ reduction */}$
 $(\lambda x. x) y = /* \beta \text{ reduction */}$
 y

b. $(\lambda x. x x y)(\lambda x. x x y) = /* \beta \text{ reduction */}$
 $(\lambda x. x x y) (\lambda x. x x y) y = /* \beta \text{ reduction */}$
 $(\lambda x. x x y) (\lambda x. x x y) y y = /* \beta \text{ reduction */}$
 $\dots /* \text{ no normal form */}$

c. $(\lambda x. x (x y)) I$
 $I (I y) (\lambda u. u) ((\lambda u. u) y) /* \beta \text{ reduction */}$
 $(\lambda u. u) y /* \beta \text{ reduction */}$
 $y /* \beta \text{ reduction */}$