

Serie 5 - Lambda Calculus

Exercise 1

Consider the following λ -expressions. Indicate which occurrences of variables are bound and which ones are free in the expressions.

1. $(\lambda a b . c d a b) a b (\lambda c d . d c) (\lambda e f . f) e$
2. $\{(\lambda u v . \lambda w . w (\lambda x . x(u)) (v)) (y)\} (\lambda z . \lambda y . z(y))$
3. $\lambda y . (\lambda x . z(x(\lambda x . y(z)))) (\lambda z . y(x(z)))$

Exercise 2

Define `true`, `false`, `and`, `or`, `not` and `xor` in Lambda Calculus.

Exercise 3

Reduce the following λ -expressions to their normal form whenever possible.

- a. $P \equiv (\lambda x . x (x y)) I$ where $I \equiv \lambda u . u$
- b. $Y \equiv \lambda f . Q Q$ where $Q \equiv (\lambda x . f(x x))$
- c. $L \equiv (\lambda x . x x y) (\lambda x . x x y)$
- d. $(\lambda x . x L) M$ where $M \equiv \lambda x . y$ and $L \equiv (\lambda x . x x y) (\lambda x . x x y)$
- e. $\{\{\lambda u v . \{\lambda w . w (\lambda x . x(u))\} (v)\} (y)\} (\lambda z . \lambda y . z(y))$