Solution Programming Language Semantics

Instructions:

Solutions of the exercises are to be delivered before Thursday, the 26th of April at 10:15AM.
Solutions should be placed in a separate folder with the name "**Assignment07**".
Please submit answers to all the exercises in **one** text file.

Exercise 1 (3 points)

Extend the abstract syntax and the semantic functions **P**, **S** and **E** of the Calculator Language, defined at the lecture hours, in order to include the possibility of subtraction and division of two expressions. In case the divider is zero, the result should be the string "NOT A NUMBER".

Answer:

% Extended syntactic and semantic rules are written in blue.

Abstract syntax:

Prog ::= 'ON' Stmt

Stmt ::= Expr 'TOTAL' Stmt
| Expr 'TOTAL' 'OFF'

Expr ::= Expr1 '+' Expr2
| Expr1 '' Expr2*
| Expr1 '-' Expr2
| Expr1 '/' Expr2
| 'IF' Expr1 ',' Expr2 ',' Expr3
| 'LASTANSWER'
| 'ERROR'
| '(' Expr ')'
| Num

Semantics:

$\boldsymbol{P}$: *Program* $\rightarrow$ *Int** $\cup$ *String*
$\boldsymbol{P}$ $[\![$ *ON S* $]\!]$ = $\boldsymbol{S}$ $[\![$ *S* $]\!]$ *(0)*

$\boldsymbol{S}$: *ExprSequence* $\rightarrow$ *Int* $\rightarrow$ *Int** $\cup$ *String*
$\boldsymbol{S}$ $[\![$ *E TOTAL S* $]\!]$ *(n)* = *let n' =* $\boldsymbol{E}$ $[\![$ *E* $]\!]$ *(n) in*
cons(n', $\boldsymbol{S}$ $[\![$ *S* $]\!]$ *(n'))*
$\boldsymbol{S}$ $[\![$ *E TOTAL OFF* $]\!]$ *(n)* = *[* $\boldsymbol{E}$ $[\![$ *E* $]\!]$ *(n)]*

$\mathbf{E} : Expression \rightarrow Int \rightarrow Int \cup String$
$\mathbf{E}\ [\![E1 + E2]\!]\ (n) = \mathbf{E}\ [\![E1]\!]\ (n) + \mathbf{E}\ [\![E2]\!]\ (n)$
$\mathbf{E}\ [\![E1 - E2]\!]\ (n) = \mathbf{E}\ [\![E1]\!]\ (n) - \mathbf{E}\ [\![E2]\!]\ (n)$
$\mathbf{E}\ [\![E1 / E2]\!]\ (n) = \text{if } \mathbf{E}\ [\![E1]\!]\ (n) = 0$
$\quad \text{then } \mathbf{E}\ [\![ERROR]\!]\ (n)$
$\quad \text{else } \mathbf{E}\ [\![E1]\!]\ (n) / \mathbf{E}\ [\![E2]\!]\ (n)$
$\mathbf{E}\ [\![ERROR]\!]\ (n) = \text{“NOT A NUMBER”}$
$\mathbf{E}\ [\![E1 * E2]\!]\ (n) = \mathbf{E}\ [\![E1]\!]\ (n) * \mathbf{E}\ [\![E2]\!]\ (n)$
$\mathbf{E}\ [\![IF\ E1, E2, E3]\!]\ (n) = \text{if } \mathbf{E}\ [\![E1]\!]\ (n) = 0$
$\quad \text{then } \mathbf{E}\ [\![E2]\!]\ (n)$
$\quad \text{else } \mathbf{E}\ [\![E3]\!]\ (n)$
$\mathbf{E}\ [\![LASTANSWER]\!]\ (n) = n$
$\mathbf{E}\ [\![(E)]\!]\ (n) = \mathbf{E}\ [\![E]\!]\ (n)$
$\mathbf{E}\ [\![N]\!]\ (n) = N$

Exercise 2 (3 points)

Consider a language of binary numbers. The number **'111'** is intended to denote the natural number **7**. Define the syntax, the semantic functions and the domain of this language. As a test evaluate **'10101'**.

Answer:

Syntax:

$Number := Digit \mid Number\ Digit$
$Digit := 0 \mid 1$

Semantic functions and the domain:

$value: Number \rightarrow Natural$
$value\ [\![Number\ Digit]\!] = 2 \times value\ [\![Number]\!] + value\ [\![Digit]\!]$
$value\ [\![0]\!] = 0$
$value\ [\![1]\!] = 1$

Test:

$$\begin{aligned}
value\ [\![10101]\!] &= 2 \times value\ [\![1010]\!] + value\ [\![1]\!] \\
&= 2 \times (2 \times value\ [\![101]\!] + value\ [\![0]\!]\ ) + value\ [\![1]\!] \\
&= 2 \times (2 \times (\ 2 \times value\ [\![10]\!] + value\ [\![1]\!]\ ) + value\ [\![0]\!]\ ) + value\ [\![1]\!] \\
&= 2 \times (2 \times (\ 2 \times (\ 2 \times value\ [\![1]\!] + value\ [\![0]\!]\ ) + value\ [\![1]\!]\ ) + value\ [\![0]\!]\ ) + value\ [\![1]\!] \\
&= 2 \times (2 \times (\ 2 \times (\ 2 \times 1 + 0\ ) + value\ [\![1]\!]\ ) + value\ [\![0]\!]\ ) + value\ [\![1]\!] \\
&= 2 \times (2 \times (\ 2 \times 2 + value\ [\![1]\!]\ ) + value\ [\![0]\!]\ ) + value\ [\![1]\!] \\
&= 2 \times (2 \times (\ 2 \times 2 + 1) + value\ [\![0]\!]\ ) + value\ [\![1]\!] \\
&= 2 \times (2 \times 5 + value\ [\![0]\!]\ ) + value\ [\![1]\!]
\end{aligned}$$

$= 2 \times (2 \times 5 + 0\ ) + value\ [\![\ 1\ ]\!]$
$= 2 \times 10 + value\ [\![\ 1\ ]\!]$
$= 2 \times 10 + 1$
$= 21$