Prof. O. Nierstrasz
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Fixed Points

- Exercises are given every week on the PL page of the SCG website (http://scg.unibe.ch/teaching/pl)
- Solutions to each assignment must be sent to **joel.niklaus@inf.unibe.ch**
- The solutions of the assignments are to be delivered before every Thursday at 11 PM. Solutions handed in later than the specified time will not be accepted. In case of serious reasons send an e-mail to **joel.niklaus@inf.unibe.ch**

Exercise (6 points)

1. We represent non-negative integers with the following Lambda expressions:

$$\begin{aligned} 0 &\equiv \lambda f \,.\, \lambda x \,.\, x \\ 1 &\equiv \lambda f \,.\, \lambda x \,.\, fx \\ 2 &\equiv \lambda f \,.\, \lambda x \,.\, f(fx) \\ &\vdots \\ n &\equiv \lambda f \,.\, \lambda x \,.\, f^n x \end{aligned}$$

   Suppose you have defined the function **if** and the operations **add**, **pred** and **isZero**. Consider the following recursive (and hence not valid) definition for the multiplication:

$$\textbf{times} = \lambda n_1 \,.\, \lambda n_2 \,.\, \textbf{if}\ (\textbf{isZero}\ n_1)\ \mathbf{0}\ (\textbf{add}\ n_2\ (\textbf{times}\ (\textbf{pred}\ n_1)\ n_2))$$

   If we abstract the name **times**, we get the new expression:

$$\mathbf{t} = \lambda f \,.\, \lambda n_1 \,.\, \lambda n_2 \,.\, \textbf{if}\ (\textbf{isZero}\ n_1)\ \mathbf{0}\ (\textbf{add}\ n_2\ (f\ (\textbf{pred}\ n_1)\ n_2))$$

   By the FP theorem we know that **(Y t)** is a non-recursive equivalent of the above **times** definition.

   The exercise (3 pts) : write down the reduction sequence to demonstrate that

$$(((\mathbf{Y}\ \mathbf{t})\ \mathbf{1})\ \mathbf{k}) \rightarrow \mathbf{k}.$$

2. We can represent lists and list operators with the following Lambda expressions:

$$\begin{aligned} \textbf{nil} &= \lambda f \,.\, true \\ \textbf{null} &= \lambda l \,.\, l\ (\lambda h \,.\, \lambda t \,.\, false) \\ \textbf{cons} &= \lambda h \,.\, \lambda t \,.\, \lambda f \,.\, fht \\ \textbf{head} &= \lambda l \,.\, l\ (\lambda h \,.\, \lambda t \,.\, h) \\ \textbf{tail} &= \lambda l \,.\, l\ (\lambda h \,.\, \lambda t \,.\, t) \end{aligned}$$

   Example: the list $[1, 2, 3]$ is represented by the λ-expression **cons 1** (**cons 2** (**cons 3 nil**)).

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The exercise (3 pts):

(a) Translate the following definition into a non-recursive form (1.5 pts):

$$\textbf{append} = \lambda\, l_1\, .\, \lambda\, l_2\, .\, \textbf{if}\ (\textbf{null}\ l_1)\ l_2\ (\textbf{cons}\ (\textbf{head}\ l_1)\ (\textbf{append}\ (\textbf{tail}\ l_1)\ l_2))$$

(b) Test your result by appending list L_2 to list L_1, which are defined below (1.5 pts):

$$L_1 = \textbf{cons 1}\ (\textbf{cons 2 nil}) \ \text{ and } \ L_2 = \textbf{cons 3 nil}$$