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Solution Language Semantics

- Exercises are given every week on the PL page of the SCG website (http://scg.unibe.ch/teaching/pl)
- Solutions to each assignment must be sent to **mohammadreza.hazhirpasand@inf.unibe.ch**
- The solutions of the assignments are to be delivered before every Thursday at 11 PM. Solutions handed in later than the specified time will not be accepted. In case of serious reasons send an e-mail to **mohammadreza.hazhirpasand@inf.unibe.ch**

Exercise (6 points)

1. Extend the abstract syntax and the semantic functions **P**, **S** and **E** of the Calculator Language, defined at the lecture hours, in order to include the possibility of subtraction and division of two expressions. In case the divider is zero, the result should be the string "NOT A NUMBER". (3 pts)

Answer:

% Extended syntactic and semantic rules are written in blue.

Abstract syntax:

Prog ::= 'ON' Stmt

Stmt ::= Expr 'TOTAL' Stmt
| Expr 'TOTAL' 'OFF'

Expr ::= Expr1 '+' Expr2
| Expr1 '' Expr2*
| Expr1 '-' Expr2
| Expr1 '/' Expr2
| 'IF' Expr1 ',' Expr2 ',' Expr3
| 'LASTANSWER'
| 'ERROR'
| '(' Expr ')'
| Num

Semantics:

$\mathbf{P} : Program \rightarrow Int^* \cup String$
$\mathbf{P}\ [\![ON\ S]\!] = \mathbf{S}\ [\![S]\!]\ (0)$

$\mathbf{S} : ExprSequence \rightarrow Int \rightarrow Int^* \cup String$
$\mathbf{S}\ [\![E\ TOTAL\ S]\!]\ (n) = let\ n' = \mathbf{E}\ [\![E]\!]\ (n)\ in$
$cons(n', \mathbf{S}\ [\![S]\!]\ (n'))$
$\mathbf{S}\ [\![E\ TOTAL\ OFF]\!]\ (n) = [\ \mathbf{E}\ [\![E]\!]\ (n)]$

$\mathbf{E} : Expression \rightarrow Int \rightarrow Int \cup String$
$\mathbf{E}\ [\![E1 + E2]\!]\ (n) = \mathbf{E}\ [\![E1]\!]\ (n) + \mathbf{E}\ [\![E2]\!]\ (n)$
$\mathbf{E}\ [\![E1 - E2]\!]\ (n) = \mathbf{E}\ [\![E1]\!]\ (n) - \mathbf{E}\ [\![E2]\!]\ (n)$
$\mathbf{E}\ [\![E1 / E2]\!]\ (n) = \text{if } \mathbf{E}\ [\![E2]\!]\ (n) = 0$
$\quad \text{then } \mathbf{E}\ [\![ERROR]\!]\ (n)$
$\quad \text{else } \mathbf{E}\ [\![E1]\!]\ (n) / \mathbf{E}\ [\![E2]\!]\ (n)$
$\mathbf{E}\ [\![ERROR]\!]\ (n) = \text{“NOT A NUMBER”}$
$\mathbf{E}\ [\![E1 * E2]\!]\ (n) = \mathbf{E}\ [\![E1]\!]\ (n) * \mathbf{E}\ [\![E2]\!]\ (n)$
$\mathbf{E}\ [\![IF\ E1, E2, E3]\!]\ (n) = \text{if } \mathbf{E}\ [\![E1]\!]\ (n) = 0$
$\quad \text{then } \mathbf{E}\ [\![E2]\!]\ (n)$
$\quad \text{else } \mathbf{E}\ [\![E3]\!]\ (n)$
$\mathbf{E}\ [\![LASTANSWER]\!]\ (n) = n$
$\mathbf{E}\ [\![(E)]\!]\ (n) = \mathbf{E}\ [\![E]\!]\ (n)$
$\mathbf{E}\ [\![N]\!]\ (n) = N$

2. Consider a language of binary numbers. The number **'111'** is intended to denote the natural number **7**. Define the syntax, the semantic functions and the domain of this language. As a test evaluate **'10100'**. (3 pts)

Answer:

Syntax:

$Number := Digit \mid Number\ Digit$
$Digit := 0 \mid 1$

Semantic functions and the domain:

$value: Number \rightarrow Natural$
$value\ [\![Number\ Digit]\!] = 2 \times value\ [\![Number]\!] + value\ [\![Digit]\!]$
$value\ [\![0]\!] = 0$
$value\ [\![1]\!] = 1$

Test:
$value\ [\![10100]\!] = 2 \times value\ [\![1010]\!] + value\ [\![0]\!]$
$= 2 \times (2 \times value\ [\![101]\!] + value\ [\![0]\!]\ ) + value\ [\![0]\!]$
$= 2 \times (2 \times (\ 2 \times value\ [\![10]\!] + value\ [\![1]\!]\ ) + value\ [\![0]\!]\ ) + value\ [\![0]\!]$
$= 2 \times (2 \times (\ 2 \times (\ 2 \times value\ [\![1]\!] + value\ [\![0]\!]\ ) + value\ [\![1]\!]\ ) + value\ [\![0]\!]\ ) + value\ [\![0]\!]$
$= 2 \times (2 \times (\ 2 \times (\ 2 \times 1 + 0\ ) + value\ [\![1]\!]\ ) + value\ [\![0]\!]\ ) + value\ [\![0]\!]$
$= 2 \times (2 \times (\ 2 \times 2 + value\ [\![1]\!]\ ) + value\ [\![0]\!]\ ) + value\ [\![0]\!]$
$= 2 \times (2 \times (\ 2 \times 2 + 1) + value\ [\![0]\!]\ ) + value\ [\![0]\!]$
$= 2 \times (2 \times 5 + value\ [\![0]\!]\ ) + value\ [\![0]\!]$

$= 2 \times (2 \times 5 + 0\ ) + value\ [\![\ 0\ ]\!]$
$= 2 \times 10 + value\ [\![\ 0\ ]\!]$
$= 2 \times 10 + 0$
$= 20$