Graph

Data Structure

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Graph

A data structure consisting of a set of vertices (V) connected by a set of edges (E) that can be used to model relationships among the objects in a collection.

$$G = (V, E) \begin{cases} V = \{V_i | i = 1, n\} \\ E_{ij} = \{(V_i, V_j) | V_i \in V, V_j \in V\} \end{cases}$$

Graphs are also referred to as networks, vertices as nodes, and edges as links.

What is it for

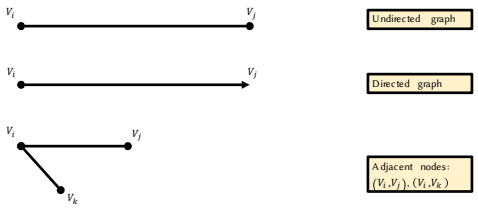
You can imagine any transportation or transmission system.

- Blood vessels, nerves, communication systems (e.g. network)

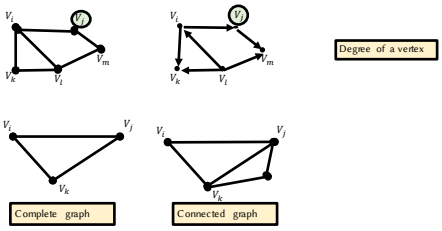
For instance, you see unique search results in Facebook based on:

- Your connections to people, places, things.
- What you're able to see on Facebook, including what your friends share with you.
- Your friends, and interests, which affect the order of your results.
- People's privacy settings. For example, if you search "photo Paris," you may see photos your friends took and shared with you first.

Graph properties



Graph properties (continue)

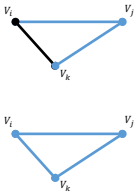


Degree of a vertex

Complete graph

Connected graph

Graph properties (continue)



Simple path

Cycle

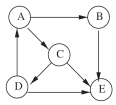
Graph implementation

```
public class Graph {
    List<Node> nodes;
}
```

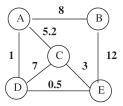
```
public class Node {

    public String name;
    public List<Edge> connections;

}
```

```
public class Edge {

    public Node start;
    public Node end;
    public double weight;
}
```


Array representation of a graph



$$G = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{pmatrix} A & B & C & D & E \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$



$$G = \begin{pmatrix} 0 & 8 & 5.2 & 1 & 0 \\ 8 & 0 & 0 & 0 & 12 \\ 5.2 & 0 & 0 & 7 & 3 \\ 1 & 0 & 7 & 0 & 0.5 \\ 0 & 12 & 3 & 0.5 & 0 \end{pmatrix}$$

Degree of a vertex in a matrix

$$G = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$Degree(V_i) = \sum_j G_{i,j} + \sum_j G_{j,i}$$

Out − degree *In − degree*

N.B. If a graph is undirected then its corresponding matrix is symmetric and for every vertex: Degree(V) = in-degree = out-degree

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Exercise

Given two vertices V_i and V_j in a graph, compute the shortest path between these two vertices with the help of the corresponding matrix of the graph.

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